Chapter 6 - Day 1

Extreme Values

The largest value a function attains on an interval is called its <u>global</u> or <u>absolute maximum value</u>.

The smallest value a function attains on an interval is called its <u>global</u> or <u>absolute minimum value</u>.

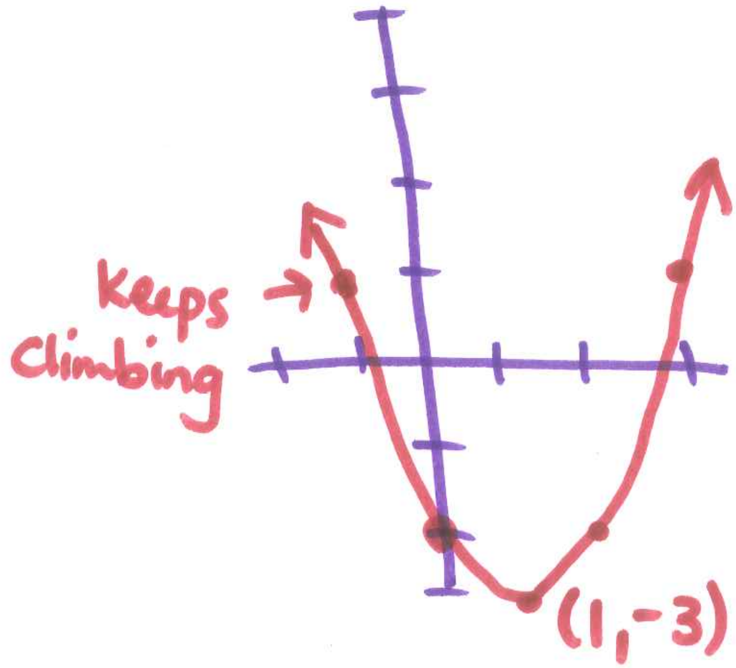
Both max and min values are called <u>global</u> or <u>absolute extreme values</u>

Ex: Find the max and min values for the function if they exist.

a) $f(x) = (x-1)^2 - 3$

minimum at $(1,-3)$

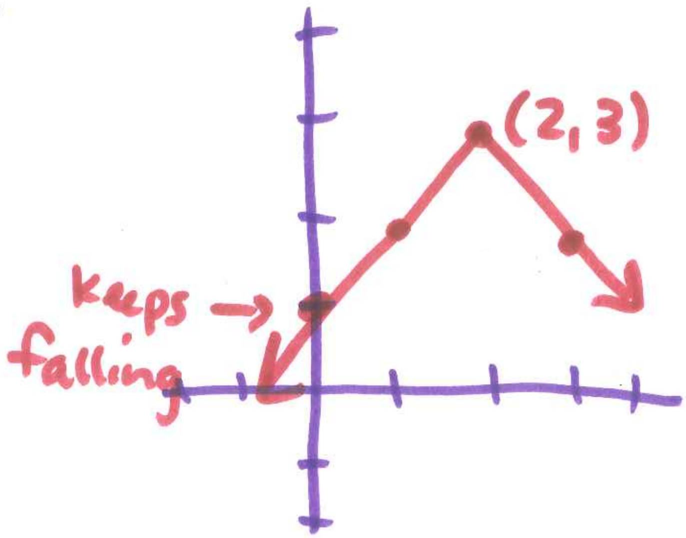
no maximum



b) $f(x) = -|x-2| + 3$

max at $(2,3)$

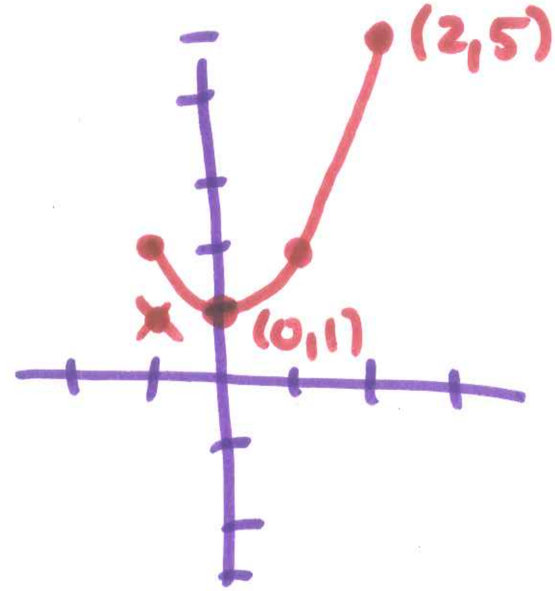
no minimum



c) $f(x) = x^2 + 1$ for $x \in [-1, 2]$

max at $(2,5)$

min at $(0,1)$



We like continuous functions over closed and bounded intervals!

Recall: Continuous means no gaps, skips, jumps. You can draw these without picking up your pencil.

An interval is closed and bounded if it has finite length and contains its endpoints.

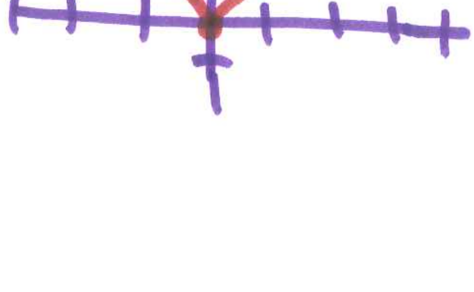
Ex: $[-2, 5]$ is closed and bounded.

Extreme Value Theorem (EVT): if

a function f is continuous on a closed, bounded interval [a,b], then the function f attains a maximum and a minimum value on [a,b].

Ex: let $f(x) = \begin{cases} \sqrt{x} & \text{for } x > 0 \\ \sqrt{-x} & \text{for } x \leq 0 \end{cases}$

Does f(x) have a max and a min on [-1, 3]?

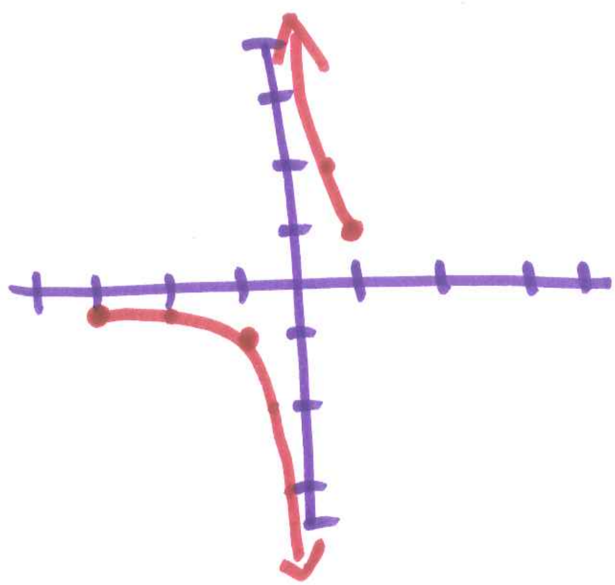


EVT says it has both!

min at (0,0)

max at $(3, \sqrt{3})$

Ex: let $g(x) = \frac{1}{x}$. Does $g(x)$ have a max and a min on $[-3, 1]$?

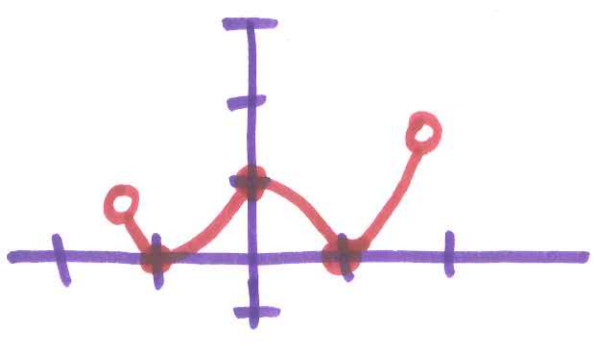


$g(x)$ is not continous so EVT does not apply.

No max

No min

Ex: let $h(x) = x^4 - 2x^2 + 1$. Does $h(x)$ have a max and a min on $(-1.25, 1.5)$?



open interval - EVT does not apply.

no max

mins at $(-1, 0)$ and $(1, 0)$